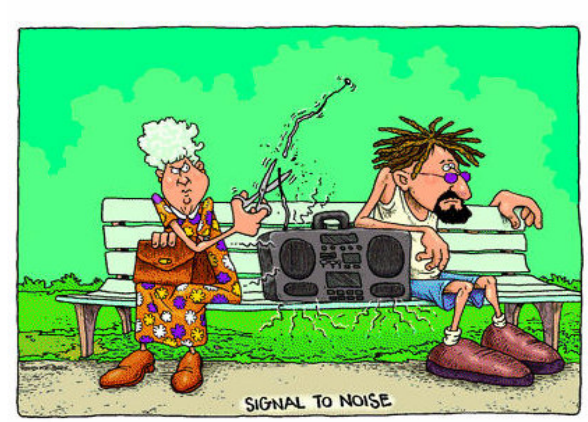
# Shot noise

We considered quantum effects in current measurements.

However, fluctuations of current (noise) give additional info about quantum system…

- What can we learn from shot noise?



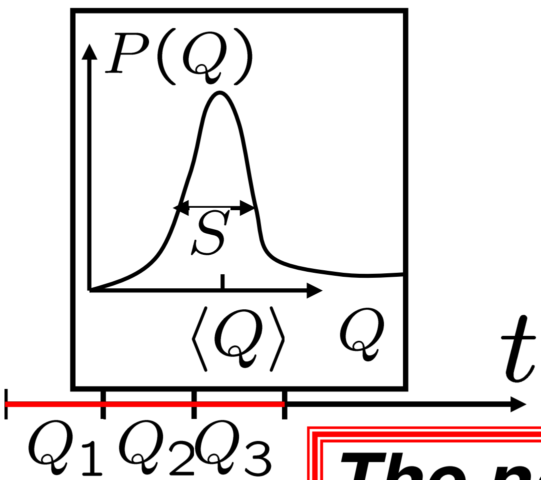
From: Beenakker and Schonenberger

- Brief history
- Shot noise and Fano factor in DC-systems
- Experiments

$$S(\omega) \propto \int \int dt dt' \left[ \langle \hat{I}(t) \hat{I}(t') \rangle - \langle \hat{I}(t) \rangle \langle \hat{I}(t') \rangle \right] e^{i\omega(t-t')}$$

# What is so interesting in noise?

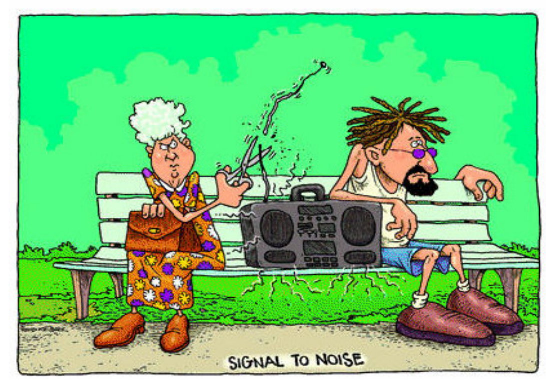
Study distribution of transmitted charge $P(Q)$ or cumulants




Charge noise

$S = \langle\langle Q^2 \rangle\rangle$

as important as current




***The noise is the signal*** (R. Landauer)

Dis**advantage**: diminish errors in measured charge

enhance $I/S$ ratio

**Advantage**: Noise is correlations

Noise is information

# Fundamental sources of noise

**Thermal fluctuations** of occupation numbers

$$\Delta n(E) = n(E) - \langle\, n(E)\,\rangle\,; \qquad f(E) = \langle\, n(E)\,\rangle$$

$$\langle(\Delta n)^2\rangle = \langle n^2\rangle - \langle\, n\,\rangle^2 = f - f^2 = f(1-f) = -(k_B T)\, df/dE$$

$\Longrightarrow$ Nyquist-Johnson equilibrium classical noise

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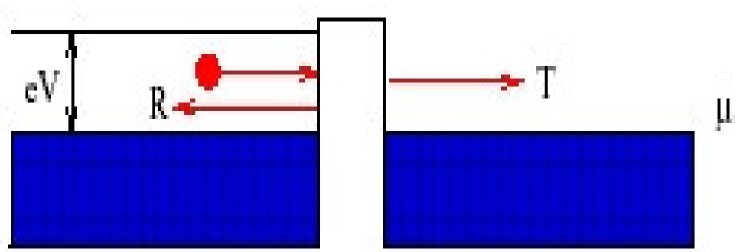
**Partition (shot) noise**




occupation numbers:

$n_I$ : incident beam

$n_T$ : transmitted beam

$n_R$ : reflected beam

averages: $\langle n_I\rangle = 1$; $\langle n_T\rangle = T$; $\langle n_R\rangle = R$;

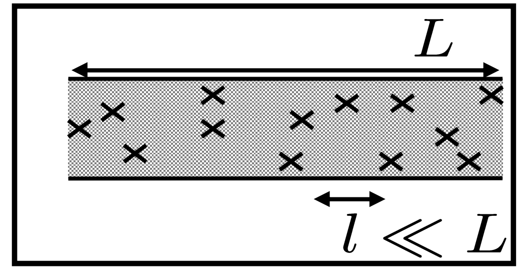
Each particle can only be either transmitted or reflected: $\langle n_T^2\rangle = 1^2 T + 0^2(1-T) = T$

$$\langle(\Delta n_T)^2\rangle = \langle n_T^2\rangle - \langle n_T\rangle^2 = \boxed{T(1-T)}$$

# Brief history:

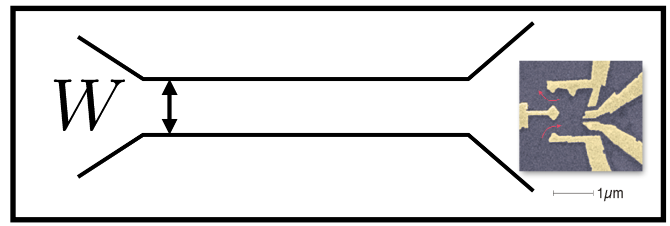
- **<80's:** Macro scale + <u>classical</u> Nyquist noise $S_{\mathrm{N}} \propto G k_{\mathrm{B}} T$

- **80's:** Meso scale + diffusive samples
  Properties of average current $I$
  No new info from noise dominated by $S_{\mathrm{N}}$



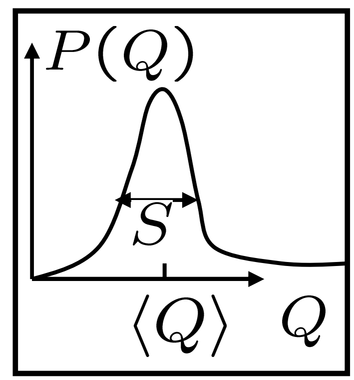

- **90's:** Ballistic Quantum Point Contacts, $N = W/(\lambda_{\mathrm{F}}/2) \sim 1$
  Ballistic quantum dots, $L \ll l$
  Shot noise at low $T$

**Theory:** full coherence, no interaction




- **00's:** AC-transport up to *GHz*, entanglement

**Theory:** FCS. Effects of interactions,decoherence
  Shot noise as a source of additional info



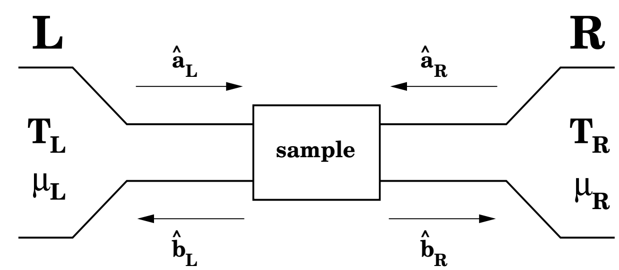

- **Future…** possible applications of shot noise?

# Current operator

Current in contact $\alpha$ $\quad \hat{I}_\alpha(t) = \frac{e}{h}\int dE\,[\hat{n}_{\alpha,in}(E,t) - \hat{n}_{\alpha,out}(E,t)]$

current amplitude: $\hat{a}_\alpha(E)$(incoming) $\hat{b}_\alpha(E)$ (outgoing)

$$\hat{I}_\alpha(t) = \frac{e}{h}\int dE' dE [\hat{a}^\dagger_\alpha(E')\hat{a}_\alpha(E) - \hat{b}^\dagger_\alpha(E')\hat{b}_\alpha(E)]e^{i(E'-E)t/\hbar}$$




# Noise spectral density

Spectral density S (noise power)

$$(1/2)\langle\langle I_\alpha(\omega)I_\beta(\omega') + I_\beta(\omega')I_\alpha(\omega)\rangle\rangle = 2\pi S_{\alpha\beta}(\omega)\delta(\omega+\omega')$$

zero-frequency limit $\qquad A(\alpha) \equiv 1_\alpha - \mathcal{S}^\dagger 1_\alpha \mathcal{S}$

$$S_{\alpha\beta} = \tfrac{e^2}{h}\int dE \sum_{mn} f_m(E) A_{mn}(\alpha)(1 - f_n(E))A_{nm}(\beta)$$

$$\boxed{S_{\alpha\beta} = \frac{e^2}{h}\int dE \ \mathrm{tr}\ \hat{f}(E)\hat{A}(\alpha)(\hat{1} - \hat{f}(E))\hat{A}(\beta)}$$

# Equilibrium noise $V = 0, k_B T \neq 0$

$$S_{\alpha\beta}(0) = \frac{e^2}{h} \int dE \ \text{tr} \ \hat{f}(E) A(\alpha)(\hat{1} - \hat{f}(E)) A(\beta)$$

Due to equilibrium, distributions are the same $\Rightarrow \hat{f} = f\hat{1}$

$$\int dE f(E)(1 - f(E)) = -k_B T \int dE \partial_E f(E) = k_B T$$

auto-correlation $\langle I_\alpha^2 \rangle$ and cross- correlations $\langle I_\alpha I_\beta \rangle$

$$\boxed{S_{\alpha\beta}(0) = (k_B T)[G_{\alpha\beta} + G_{\beta\alpha}]}$$

$$\sum_\alpha S_{\alpha\beta}(0) = \sum_\beta S_{\alpha\beta}(0) = 0$$

$$S(\omega) \approx S(0), \ \hbar\omega \ll \ \text{min} \ \{k_B T, \hbar/\tau_\text{d}\}$$

Classical Nyquist noise measures conductance

# Partition (shot) noise

$$S_{\alpha\beta}(0) = \frac{e^2}{h}\int dE \mathrm{tr}\ \hat{f}(E)A(\alpha)(\hat{1} - \hat{f}(E))A(\beta)$$

$$A(\alpha) \equiv 1_\alpha - \mathcal{S}^\dagger 1_\alpha \mathcal{S} \Rightarrow \sum_\alpha S_{\alpha\beta}(0) = \sum_\beta S_{\alpha\beta}(0) = 0$$

If $k_B T = 0, V \neq 0$ distributions are step-like

$$\int dE f(E - eV)(1 - f(E)) = eV$$

$$S = \left\| \begin{matrix} \hat{r} & \hat{t} \\ \hat{t}' & \hat{r}' \end{matrix} \right\|$$

$$S_{11}|_{N=1} = \frac{e^2}{h} eV A_{12}(1) A_{21}(1) = \frac{e^3 V}{h} r^* t t^* r = \frac{e^3 V}{h} T(1 - T)$$

$$\boxed{S = \frac{e^2}{h} eV \sum_n T_n(1 - T_n)}$$

In classical wire $T \propto l/L \ll 1$, noise is Poissonian:
Electrons appear independently with rate defined by current

$$P(N) = \frac{e^{-\lambda}\lambda^N}{N!}, \lambda = I,\ \ \langle\langle N^n \rangle\rangle = \lambda$$

$$S_S \approx (e^3 V/h) \sum T = eI$$

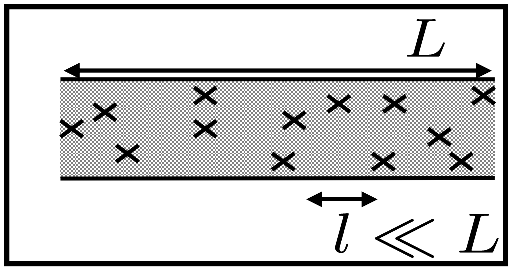
$$\boxed{S/eI = 1}$$

# Fano factor

At low $k_B T \ll eV$ we introduce **Fano factor** $\mathcal{F}$

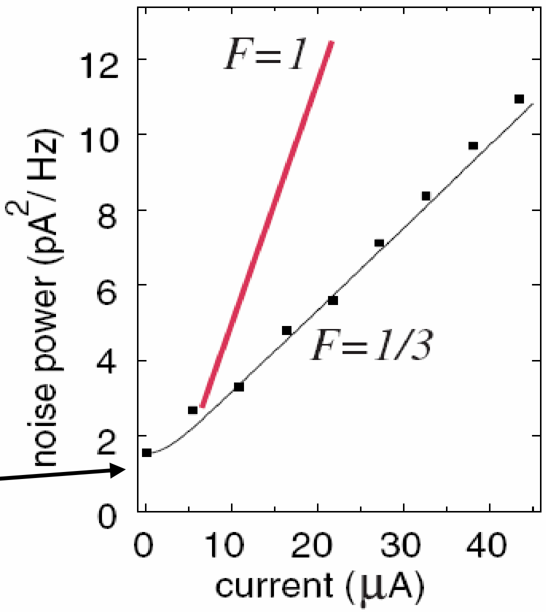
$$\mathcal{F} = \langle S/eI \rangle = \langle \sum T(1-T)/\sum T \rangle$$

Classical $\mathcal{F} = 1$ was expected in diffusive wires with many impurities




Fano factor was found very close to 1/3 (**SURPRISE?**)

At low voltages Nyquist noise becomes important and noise saturates at non-zero value

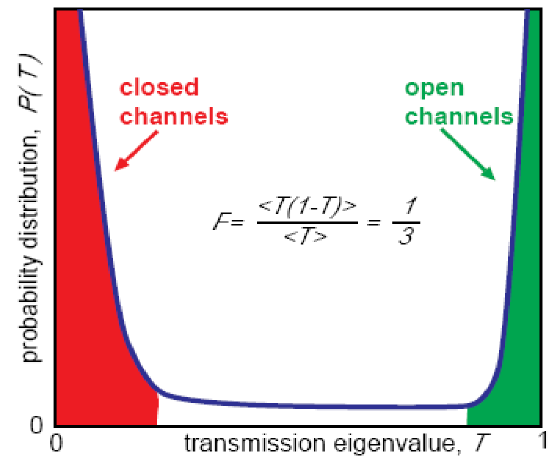


# Fano factor in wires and QPC

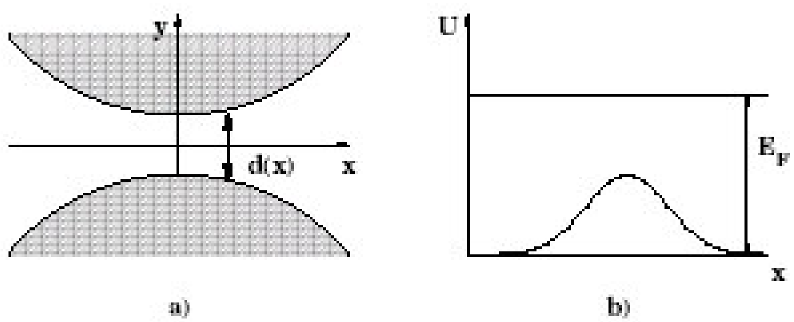
$$\mathcal{F} = \langle S/eI \rangle = \langle \sum T(1-T) / \sum T \rangle$$

Contrary to naïve ideas: existence of open channels with <u>perfect transmission +almost noiseless</u>

$$\mathcal{F} = 1/3$$



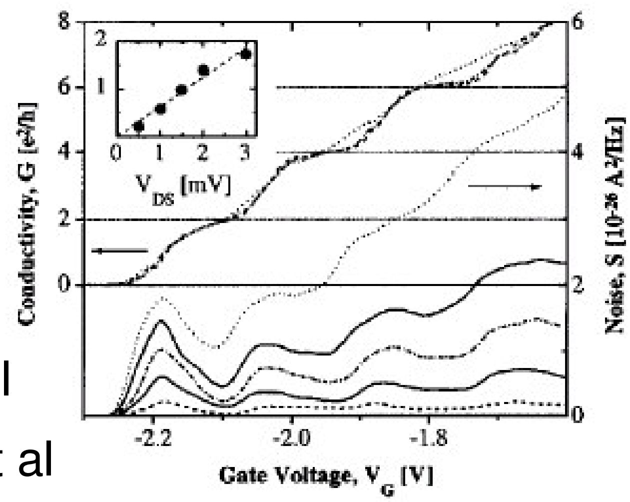

$$P(T) \propto 1/(T\sqrt{1-T}),\ \text{wires with } N \gg 1$$



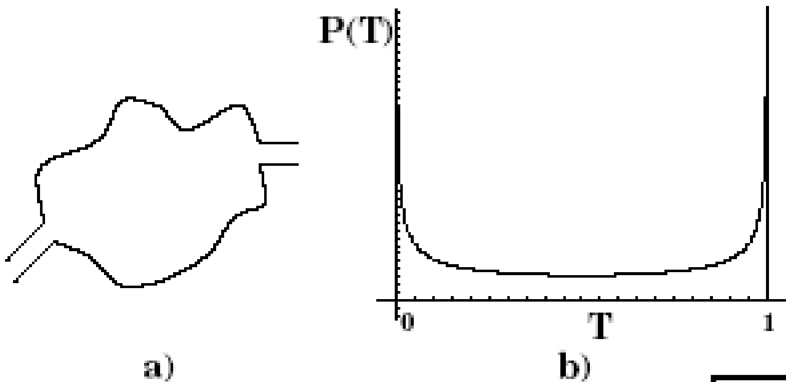

$$S = 2\frac{e^2}{h}|eV| \sum_n T_n(1-T_n)$$

Kumar et al

Reznikov et al



# Fano factor in quantum dots

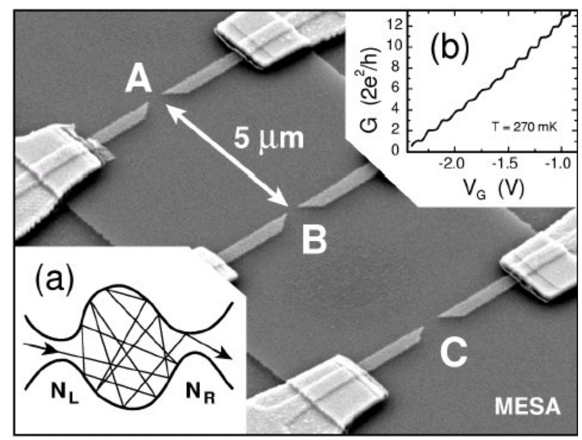



$$G = \frac{e^2}{h}\sum_n T_n$$

$$S = 2\frac{e^2}{h}|eV|\sum_n T_n(1-T_n)$$

$$P(T) \propto 1/\sqrt{T(1-T)} \Rightarrow \boxed{\mathcal{F} = \frac{N_1 N_2}{N^2} \qquad N_1 = N_2 \gg 1}$$

$$P(T) \propto T^{-1+\beta/2}, \beta = 1(2) \text{ for } B = 0(\neq 0)$$
$$\Rightarrow \mathcal{F} = \tfrac{2}{5}(\tfrac{1}{3})$$




Oberholzer et al

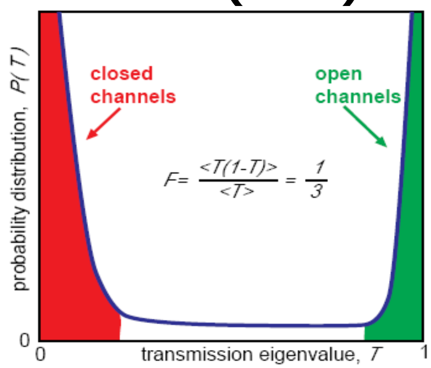
Interesting: Fano factor depends on magnetic field (quantum-mechanical result!)

# Noise: Conclusions

- Shot noise is as important as current, gives new information about properties of the system

$$\mathcal{F} = \langle S/eI \rangle = \langle \sum T(1-T) / \sum T \rangle$$

- Fano factor in universal and depends only on geometry and presence of magnetic field
- Usually Fano factor is $\mathcal{F} < 1$ due to bimodal distribution of transmission eigenvalues $P(T)$



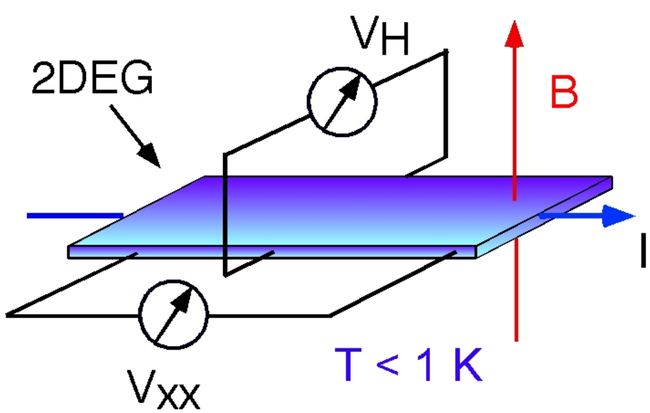
# Integer quantum Hall effect (IQHE)

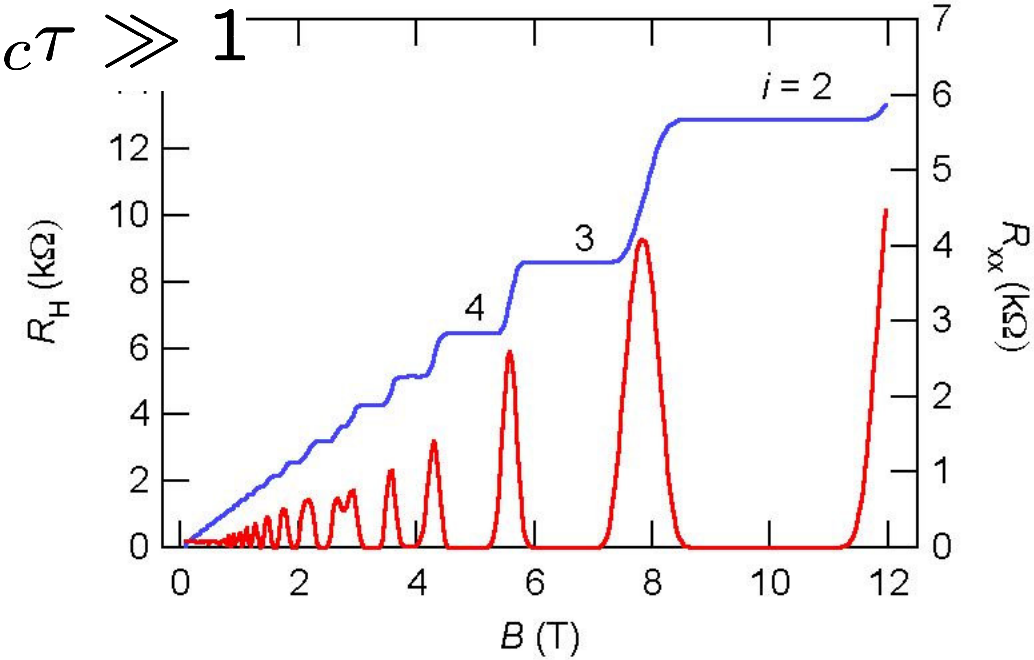
- Conducting channels were useful for quantum transport without magnetic field. What about IQHE?
- We considered effects in classically weak fields, $\omega_c \tau \ll 1$

$$l_{\mathrm{cycl}} = \frac{p_{\mathrm{F}}}{eB} \gg l_B = \sqrt{\frac{\hbar}{eB}} >\sim l \gg \lambda_{\mathrm{F}} = \frac{h}{p_{\mathrm{F}}}$$

- When $B\lambda_{\mathrm{F}}^2 \sim \Phi_0 \Leftrightarrow l_m \sim l_{\mathrm{cycl}} \sim \lambda_{\mathrm{F}}$

$$\hbar\omega_c = \frac{\hbar e B}{m} \sim E_{\mathrm{F}} \Rightarrow \omega_c \tau \gg 1$$



# Sub-band depopulation

- Low fields vs high fields: conductance is defined by the number of filled subbands

$$G = (e^2/h)N$$

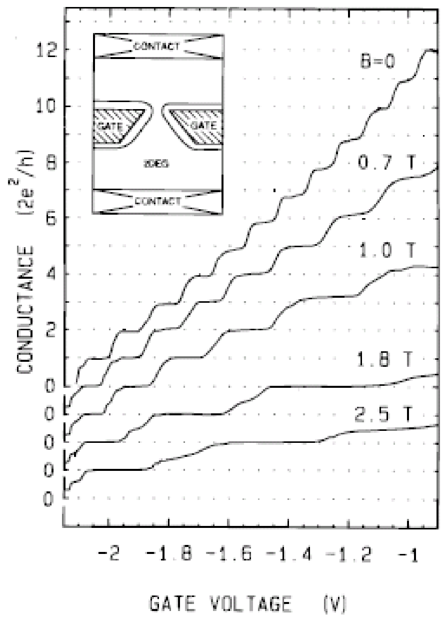



FIG. 48 Point contact conductance (corrected for a background resistance) as a function of gate voltage for several magnetic field values, illustrating the transition from zero-field quantization to quantum Hall effect. The curves have been offset for clarity. The inset shows the device geometry. Taken from B. J. van Wees et al., Phys. Rev. B. 38, 3625 (1988).

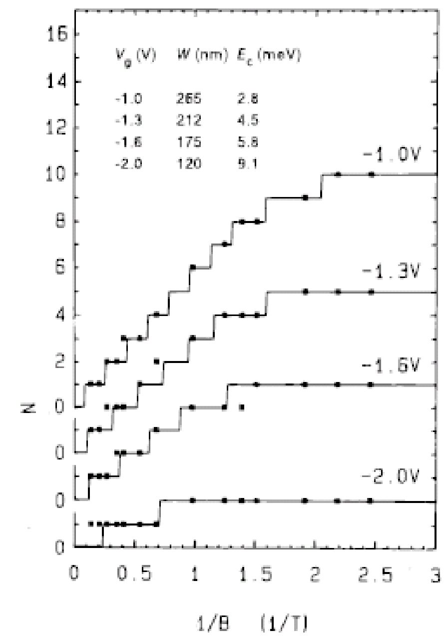
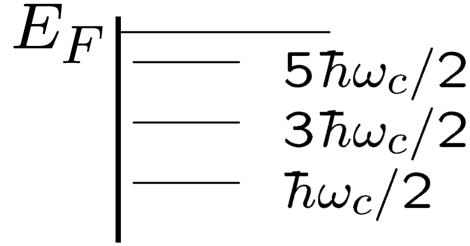


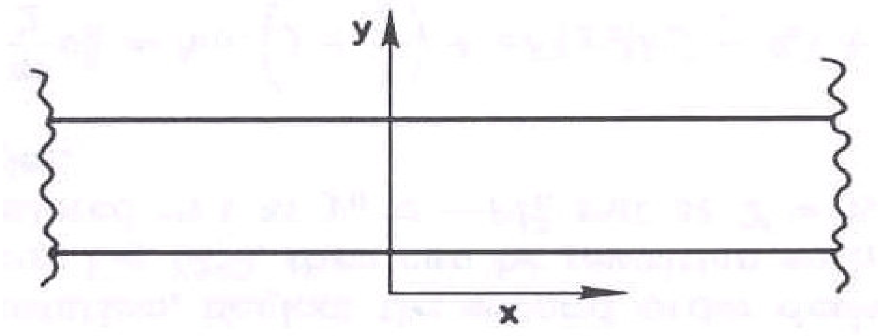

FIG. 49 Number of occupied subbands as a function of reciprocal magnetic field for several values of the gate voltage. Data points have been obtained directly from the quantized conductance (Fig. 48); solid curves are calculated for a square-well confining potential of width $W$ and well bottom $E_c$ as tabulated in the inset. Taken from B. J. van Wees et al., Phys. Rev. B 38, 3625 (1988).

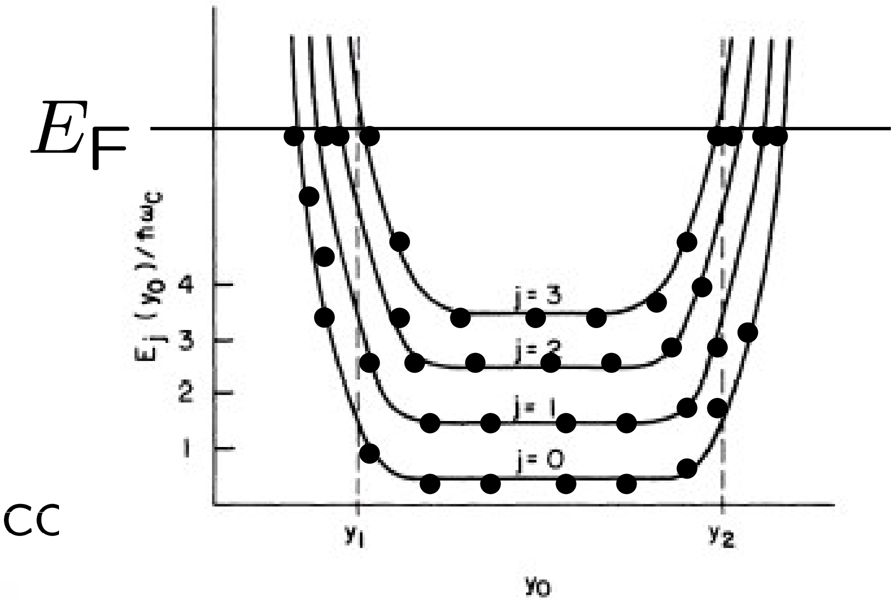
- Low fields: quantization of transversal motion $N = [k_F W/\pi]$
- High fields: Landau levels

$$N = [E_F/\hbar\omega_c + 1/2]$$



# Integer Quantum Hall




$A_x = -B_z y, A_y = 0 \Rightarrow p_x = k =$cc

$$E = \frac{p_y^2 + (\hbar k + eBy)^2}{2m} + V(y)$$

$$\Psi(\mathbf{r}, E) = e^{\imath kx}\chi(y)$$

$$E = \frac{p_y^2}{2m} + \frac{1}{2} m\omega_c^2 (y - y_{0,k})^2 + V(y)$$

In the bulk $V(y) =$const energy is independent of $y_{0,k} = -k\, l_B^2$

$$\Rightarrow E_n = \hbar\omega_c(n + 1/2)$$

# Bulk and edge IQHE in clean system

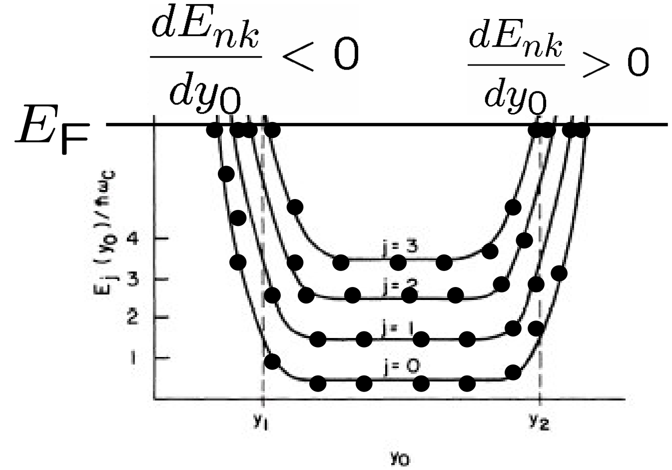
- For finite lengths $L_x, k = K\pi/L_x$ is quantized

  $y_{0,k} = -k\, l_B^2$

- $n$-th level is highly degenerate:

  $L_y L_x/\pi l_B^2 = 2AB/\Phi_0$

  $$E_{nk} = \hbar\omega_c(n + 1/2)$$




- Potential $V(y)$ lifts energies of states close to edges. Longitudal velocity depends on *n* and *k*

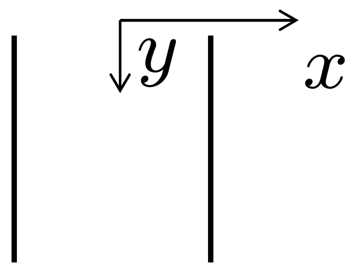
  $$v_{nk} = \hbar^{-1}\frac{dE_{nk}}{dy_0}\frac{dy_0}{dk}$$

- Each bulk Landau level brings up two oppositely moving narrow edge channels

# Classification of orbits

- Orbits: cyclotron, skipping orbits and transversal paths (0,1,2 bounces)

$$X = -k\, l_B^2 = const$$



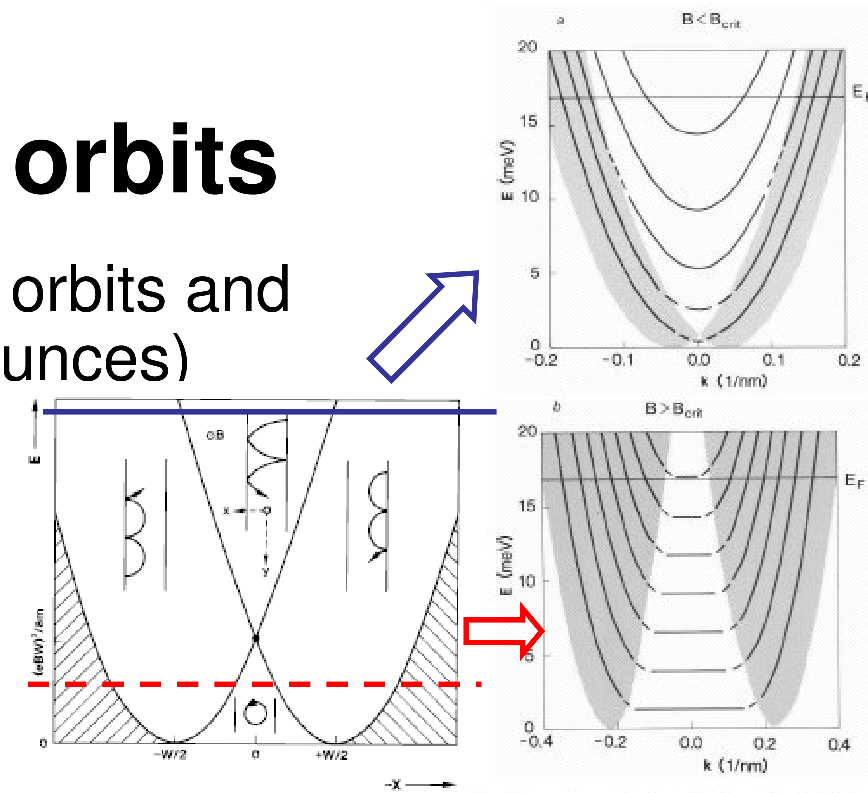



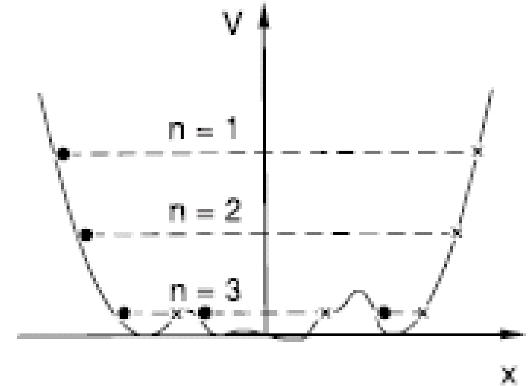

FIG. 40 Dispersion relation $E_n(k)$, calculated for parameters: (a) $W = 100$ nm, $B = 1$ T; (b) $W = 200$ nm, $B = 1.5$ T. The horizontal line at 17 meV indicates the Fermi energy. The shaded area is the region of classical skipping orbits and is bounded by the two parabolas shown in Fig. 39 (with the correspondence $k = -XeB/\hbar$). Note that in (a) edge states coexist at the Fermi level with states interacting with both boundaries ($B < B_{crit} \equiv 2\hbar k_F/eB$), while in (b) all states at the Fermi level interact with one boundary only ($B > B_{crit}$). Taken from C. W. J. Beenakker et al., Superlattices and Microstructures 5, 127 (1989).

- Small fields: only traversal trajectories
- Large fields $B > B_{crit}$

regions of edge states (extended) moving in different directions are separated by region of cyclotron orbits (localized)

# Edge states: smooth potential

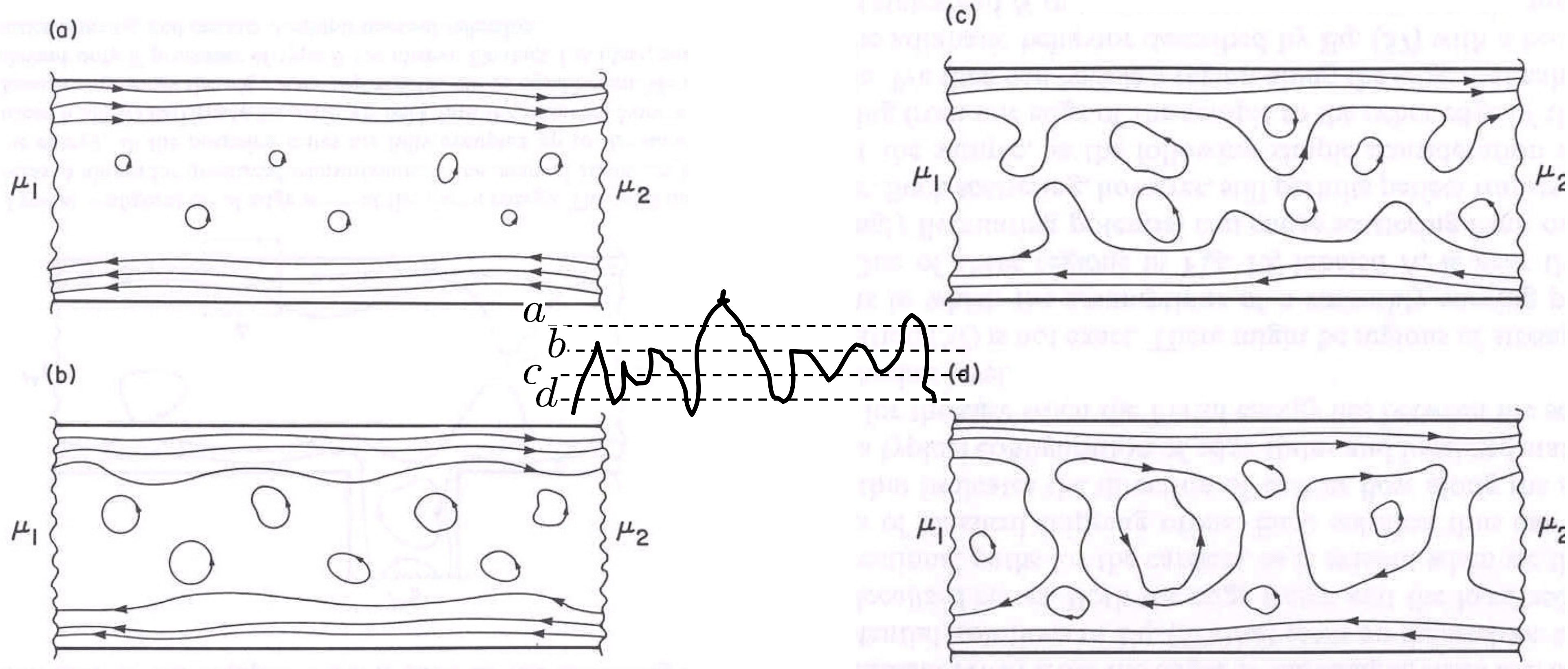
$$l_{\text{cycl}} |\nabla V(x,y)| << \hbar\omega_c$$




$$\vec{F} \propto [\nabla V \times \vec{B}] \Rightarrow \vec{v} \perp \vec{E}$$

velocity $v = cE(x,y)/B$

Transition from N=3 to N= 2 edge states

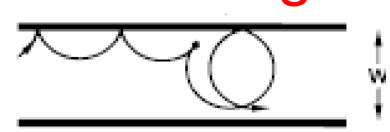


# Waveguides vs IQHE:

- Spatial confinement of channels:
  transversally extended vs localized on edge
- Sensitivity of conductance quantization to disorder:

destroyed by little disorder vs robustness to scattering

(B-field suppresses backscattering)



- Separation of channels

Only in k-space (adiabaticity needs slowly varying contact)
  vs spatially separated channels on one edge

- Channels equilibriate on

mesoscopic $L_\phi$ vs macroscopic scale $> 0.1\ \mathrm{mm}$

(no conductivity tensor!)

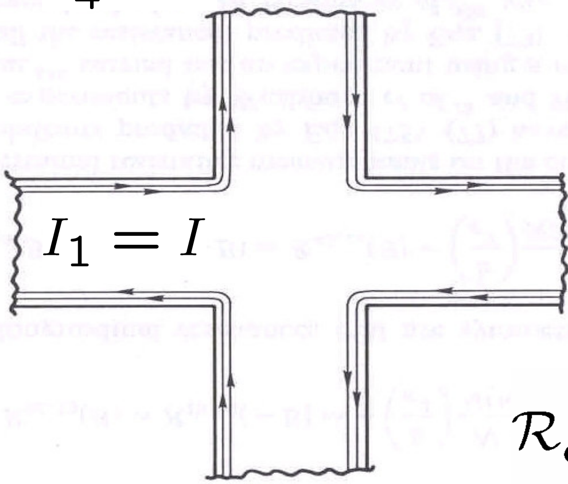
Properties of contacts (ideal/ non-ideal) are important,
  get back to Landauer-Büttiker approach!

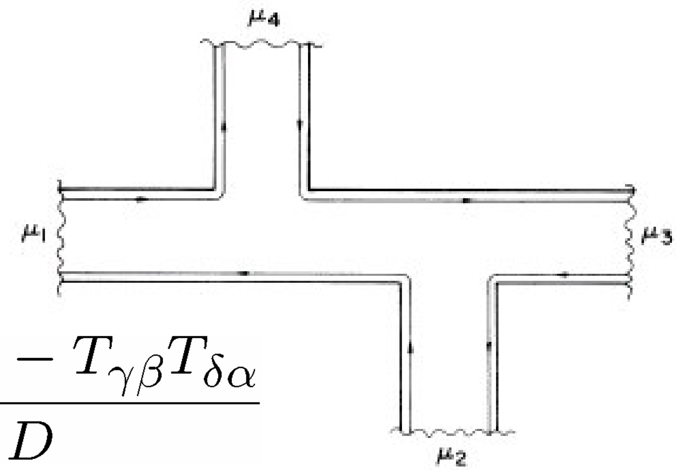
# Simple IQHE

assumed ideal contacts

$V_4 = 0$

$I_1 = I$




$$\mathcal{R}_{\alpha\beta,\gamma\delta} = \frac{h}{e^2}\frac{T_{\gamma\alpha}T_{\delta\beta} - T_{\gamma\beta}T_{\delta\alpha}}{D}$$

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix} = \frac{e^2}{h}\left(\begin{array}{ccc|c} N & -N & 0 & 0 \\ 0 & N & -N & 0 \\ 0 & 0 & N & -N \\ \hline -N & 0 & 0 & N \end{array}\right)\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \frac{Ne^2}{h}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$(Ne^2/h)V_2 = 0 \cdot I + 1 \cdot 0 + 1 \cdot (-I)$ Hall

$(Ne^2/h)(V_1 - V_2) = (I_1 + I_2 + I_3) - (I_2 + I_3)$ 2t

$$R_L = R_{12,34} = 0, \quad R_H = R_{13,24} = \frac{h}{e^2}\frac{1}{N} = R_{2t}$$

# Selective scattering: QPC+IQHE

- Potential created by QPC selectively transmits $M < N$ channels

$$\begin{pmatrix} I_s \\ I_d \\ I_l \\ I_r \end{pmatrix} = \frac{e^2}{h} \begin{pmatrix} N & -M & M-N & 0 \\ 0 & N & 0 & -N \\ -N & 0 & N & 0 \\ 0 & M-N & -M & N \end{pmatrix} \begin{pmatrix} V_s \\ V_d \\ V_l \\ V_r \end{pmatrix}$$

$$\begin{pmatrix} N & M & N-M \\ 0 & M & 0 \\ N & M & N \end{pmatrix} \begin{pmatrix} I \\ -I \\ 0 \end{pmatrix} = \frac{e^2 MN}{h} \begin{pmatrix} V_s \\ V_d \\ V_l \end{pmatrix}$$

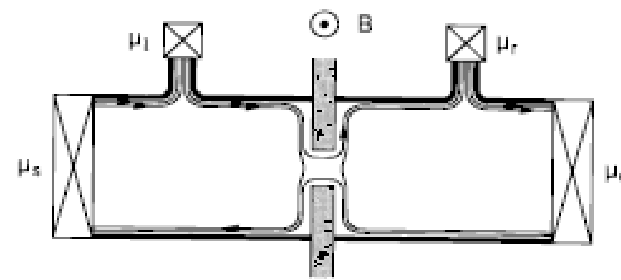
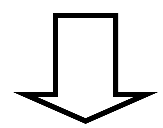



FIG. 81 Motion along equipotentials in the QHE regime, in a four-terminal geometry with a saddle-shaped potential formed by a split gate (shaded). Ideal contacts are assumed.

$$R_L = R_{sd,lr} = \frac{h}{e^2}\left(\frac{1}{M} - \frac{1}{N}\right)$$

$$R_{2t} = R_{sd,sd} = \frac{h}{e^2}\frac{1}{M}$$

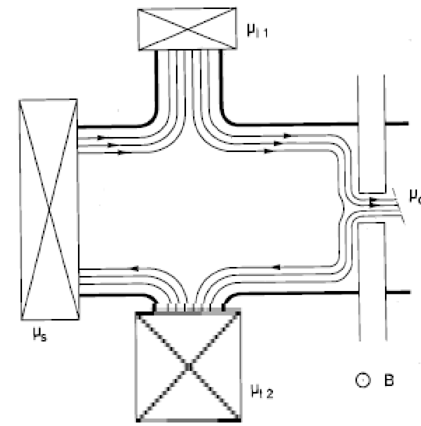
$$R_H = \frac{h}{e^2}\frac{1}{N}$$



# QHE: Non-ideal contacts

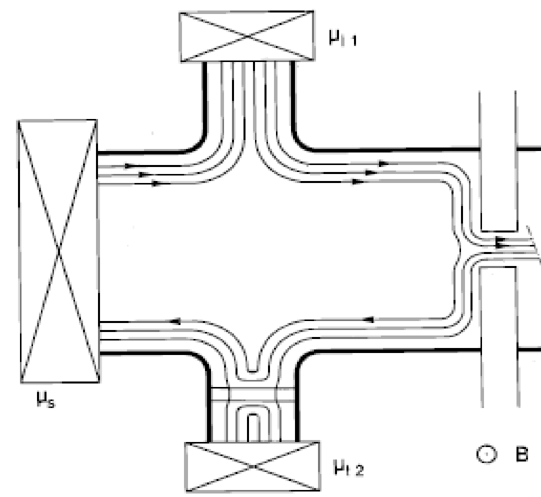
$$\begin{pmatrix} I_s \\ I_d \\ I_1 \\ I_2 \end{pmatrix} = \frac{e^2}{h} \begin{pmatrix} -N & 0 & 0 & N \\ 0 & -M & M & 0 \\ N & 0 & -N & 0 \\ 0 & M & N-M & -N \end{pmatrix} \begin{pmatrix} V_s \\ V_d \\ V_1 \\ V_2 \end{pmatrix}$$

$$\mathcal{R}_{\alpha\beta,\gamma\delta} = \frac{h}{e^2} \frac{T_{\gamma\alpha}T_{\delta\beta} - T_{\gamma\beta}T_{\delta\alpha}}{D} \qquad R_H = R_{sd,12} = \frac{h}{e^2}\frac{1}{N}$$




Channels do not interact on the same edge ⟹ no equilibrium

Ideal contacts ⟹ all channels carry the same current (equilibrium due to reservoirs), quantization is perfect




If contact is disordered, channel is reflected and exact quantization is violated!

**Conclusions: L.-B. approach is useful in IQHE**

# AC transport and interactions

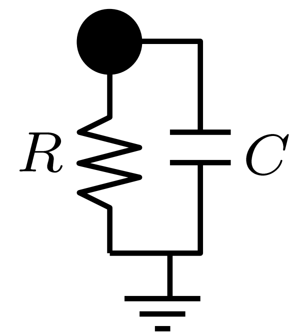
- We neglected Coulomb interactions. Non-interacting theory works in DC with strong Coulomb interaction
- 2D poorly screens additional charges, unlike 3D
- In AC-measurements in 2D have to consider nearby conductors, gates etc (finite impedance of capacitors!)

$$Z_C = -i/\omega C \neq \infty, \ \omega \neq 0$$

- Next: consider AC-transport through
- Quantum dots
- Samples in IQHE regime

# Necessary estimates for 2D

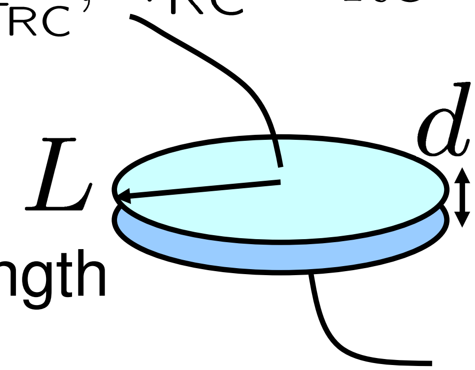
- At finite frequencies electrons absorb quanta, so energy dependence of S-matrix is important
  $$\Delta E \sim \hbar/\tau_{\mathrm{d}} \sim \hbar\cdot\mathrm{MHz}\ \mathrm{(dots)}$$
- Capacitive interaction with conductors modifies current at finite $\omega\tau_{\mathrm{RC}} \sim 1$




$$Z = \frac{R}{1+i\omega\tau_{\mathrm{RC}}}, \quad \tau_{\mathrm{RC}} = RC$$

$$R \sim h/Ne^2, C \sim \varepsilon\ \mathrm{max}\ \{L, L^2/d\}$$

$$\tau_{\mathrm{RC}} = RC \sim \frac{a_B}{v_{\mathrm{F}}} \cdot\ \mathrm{max}\ \left(1, \frac{L}{d}\right)$$




$a_B = \frac{\varepsilon\hbar^2}{m^* e^2} <\sim \lambda_{\mathrm{F}}\ \mathrm{(2D)}$ analog of 3D screening length

In result, $\tau_{\mathrm{RC}}$ is important at <GHz

In IQHE exact quantization of $G$ is destroyed (bad for metrology)

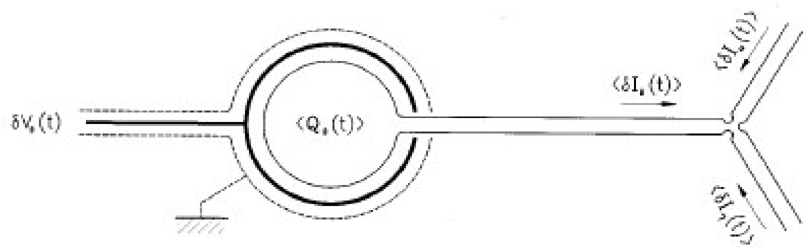
# Dynamic potentials

- Capacitive interaction with conductors leads to change of internal potential $U(\vec{r},t)$ $\Longrightarrow$ displacement currents are important $\Longrightarrow$ self-consistently define $U(\vec{r},t)$

- Key idea: total current = sum of particle and displacement currents $\nabla \times \vec{H} = \frac{4\pi}{c}\vec{j}_p + \frac{1}{c}\partial_t \vec{E}$

External source: potentials applied to terminals
Internal response: self-consistent electric potential

$$dV_\alpha(t) = dV_\alpha(\omega)e^{-i\omega t} \qquad dU(\omega,\mathbf{r})e^{-i\omega t}$$



# Non-interacting AC-response

$$I_\alpha(\omega) = \sum_\beta G_{\alpha\beta} V_\beta(\omega)$$

$$G_{\alpha\beta}(\omega) = \frac{dI_\alpha(\omega)}{dV_\beta(\omega)} = \frac{e^2}{h}\int dE \,\mathrm{tr}\,\left(\hat{A}(\alpha; E, E+\hbar\omega)1_\beta\right)\frac{f_\beta(E) - f_\beta(E+\hbar\omega)}{\hbar\omega}$$

$$\hat{A}(\alpha; E, E') = 1_\alpha - S^\dagger(E)1_\alpha S(E')$$

- In the limit $\omega \longrightarrow 0$ reproduces DC but
  - Problem: charge is not conserved

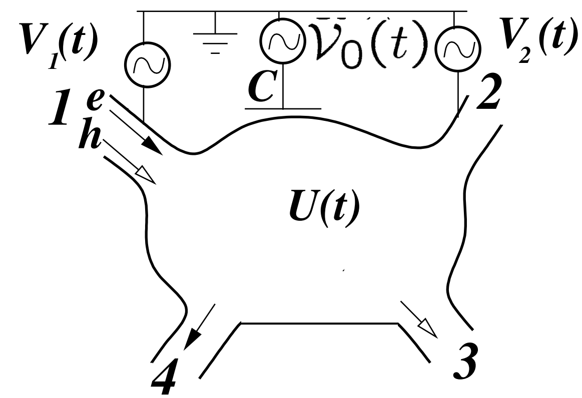
$$\sum_\alpha G_{\alpha\beta}(\omega) \neq 0 \Leftrightarrow 1 - S^\dagger(E)S(E+\hbar\omega) \neq 0$$

- Need to solve problem self-consistently: incoming charges lead to change in internal potential $U(\omega)$ It affects incoming currents

- A term due to $U(\omega)$ exists in $I_\alpha(\omega)$

# Self-consistent solution

- Non-interacting (partilce) current $I_\alpha(\omega) = \sum_\beta G_{\alpha\beta} V_\beta(\omega)$ is supplemented by a feedback $\chi_\alpha(\omega) U(\omega)$ of internal potential (displacement current)




$$\boxed{I_\alpha(\omega) = \sum_\beta G_{\alpha\beta} V_\beta(\omega) - \chi_\alpha U(\omega)}$$

- Gauge invariance

$$\Rightarrow [\sum_\beta G_{\alpha\beta} - \chi_\alpha]\delta V = 0$$

$$\frac{dq}{dt} = \sum_\alpha I_\alpha(t) \Rightarrow i\omega \sum_j C_j (U(\omega) - \mathcal{V}_j(\omega)) = \sum_\alpha I_\alpha(\omega)$$

- Charge conservation:

$$i\omega \sum_j C_j [U(\omega) - \mathcal{V}_j(\omega)] = \sum_{\alpha\beta} G_{\alpha\beta} [V_\beta(\omega) - U(\omega)]$$

# Self-consistency for single gate $\mathcal{V}_0$

$$U = \frac{i\omega C\mathcal{V}_0 + \sum_{\gamma\delta} G_{\gamma\delta}V_\delta}{i\omega C + \sum_{\gamma\delta} G_{\gamma\delta}} \rightarrow \begin{cases} \mathcal{V}_0 & C \rightarrow \infty \\ \frac{\sum_{\gamma\delta} G_{\gamma\delta}V_\delta}{\sum_{\gamma\delta} G_{\gamma\delta}} & C \rightarrow 0 \end{cases}$$

$$\boxed{I_\alpha = \sum_\beta \mathcal{G}_{\alpha\beta}V_\beta - \mathcal{X}_\alpha\mathcal{V}_0} \rightarrow \sum_\beta G_{\alpha\beta}(V_\beta - \mathcal{V}_0), \ C \rightarrow \infty$$

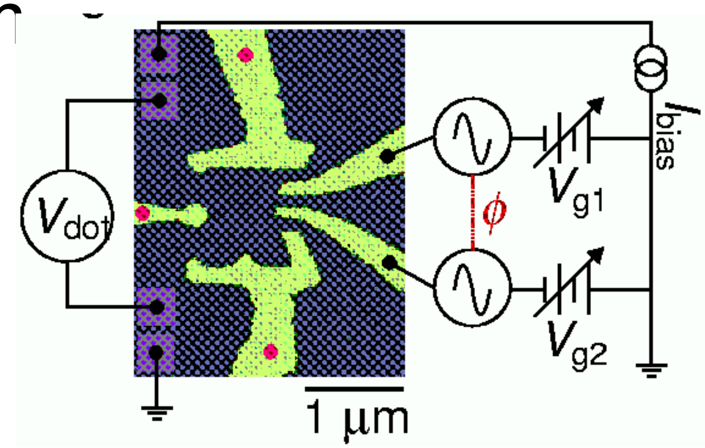
non-interacting limit

$$\mathcal{G}_{\alpha\beta} = G_{\alpha\beta} - \frac{\sum_\gamma G_{\alpha\gamma}\sum_\gamma G_{\gamma\beta}}{i\omega C + \sum_{\gamma\delta} G_{\gamma\delta}} \rightarrow \begin{cases} G_{\alpha\beta} & C \rightarrow \infty \\ G_{\alpha\beta} - \frac{\sum_\gamma G_{\alpha\gamma}\sum_\gamma G_{\gamma\beta}}{\sum_{\gamma\delta} G_{\gamma\delta}} & C \rightarrow 0 \end{cases}$$

$$\mathcal{X}_\alpha = \frac{i\omega C\sum_\gamma G_{\alpha\gamma}}{i\omega C + \sum_{\gamma\delta} G_{\gamma\delta}} \rightarrow \begin{cases} \sum_\gamma G_{\alpha\gamma} & C \rightarrow \infty \\ 0 & C \rightarrow 0 \end{cases}$$

# Examples of AC-biased systems:

- *Internally* ac-biased dot: quantum pump creates **non-zero dc-current.**




Can we kill noise and reach

$$I/S \to \infty \ ?$$

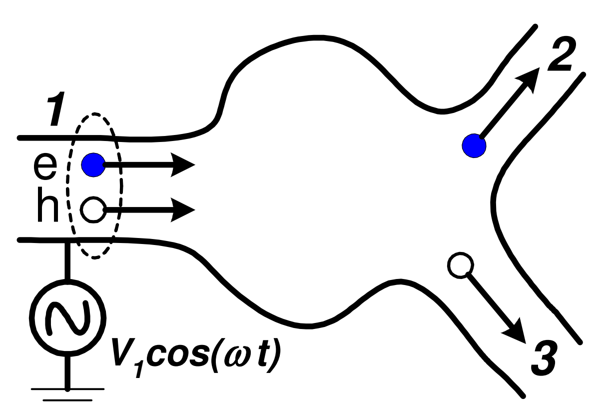
- *Externally* ac-biased dot

Noise is a detector of

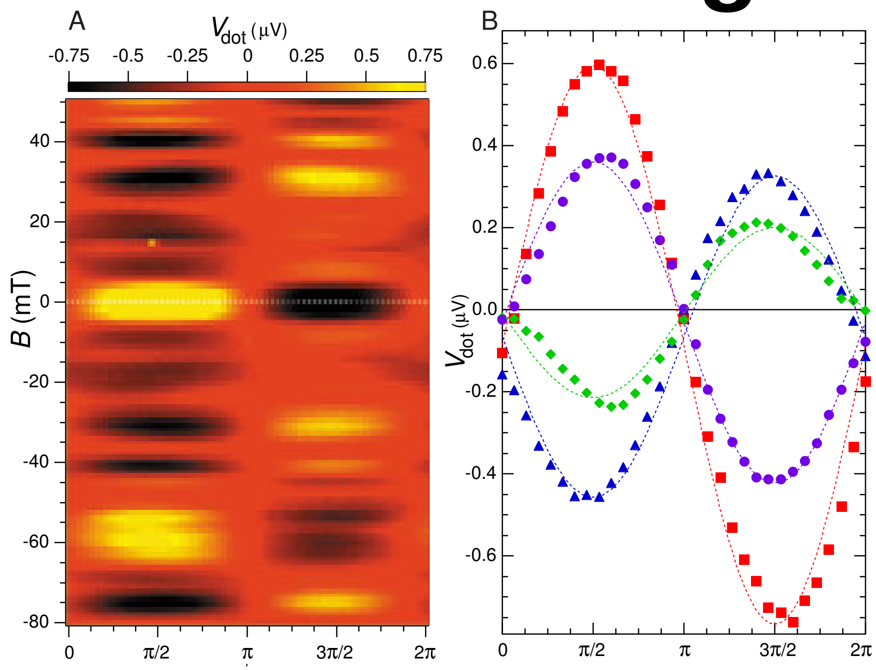
✓ quantum-mechanical correlations,

✓ interactions,

✓ decoherence

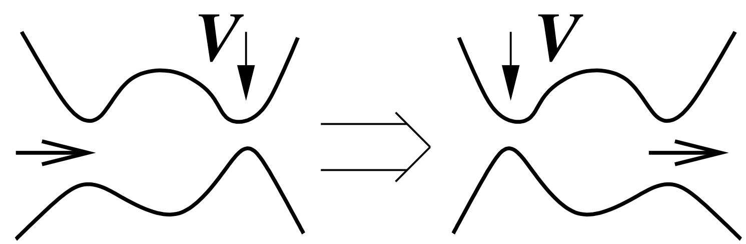


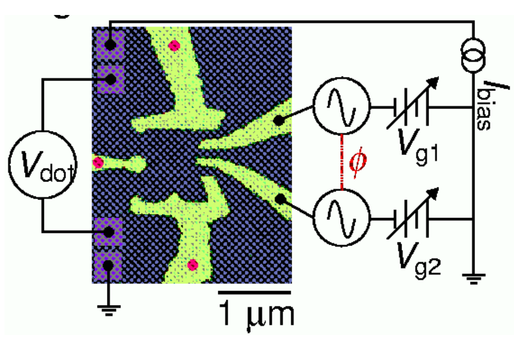
# Noise through quantum pumps




Switkes et al.'99

**Out-of-phase voltages pump electron wave function from one reservoir to the other**

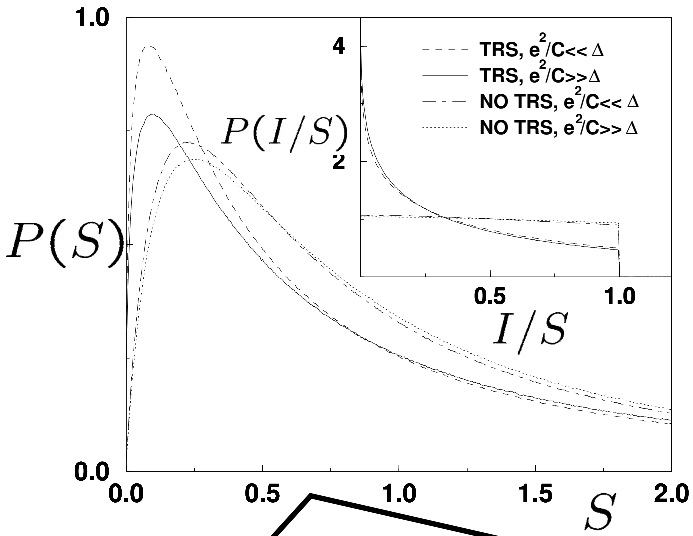


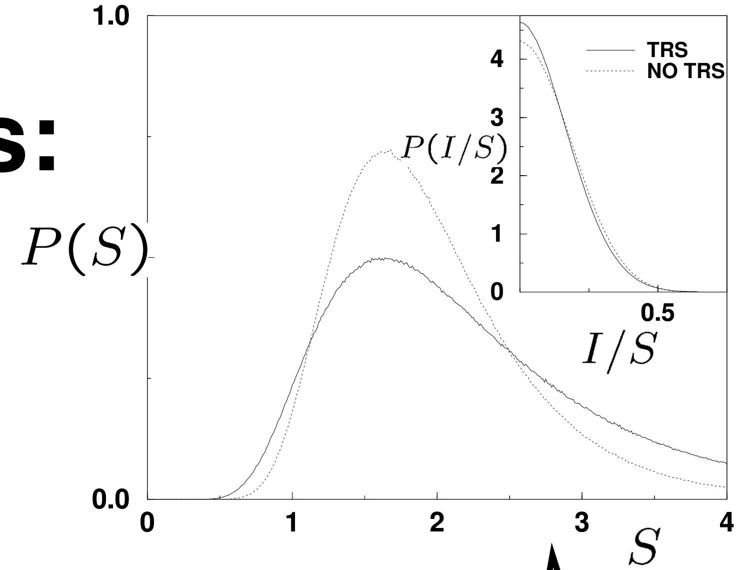




DC-current can have any sign, but is zero on average (Brouwer'98)

**Q: Can a noiseless pump produce current?**

# Results:




Single-channel dot $N_L = N_R = 1$

$P(S)$ highly non-Gaussian, modified by interaction

$I/S < 1$ is limited and $\mathcal{F} = S/I > 1$

**super-Poissonian!**

Electron pump is super- Poissonian $\mathcal{F} > 1$

Noise has non-Gaussian distribution $P(S)$

$P(S)$ is narrow for wide contacts, $N \gg 1$

Multi-channel dot

$N_L = N_R = 5$

$P(S)$ is closer to Gaussian

no effect of interactions

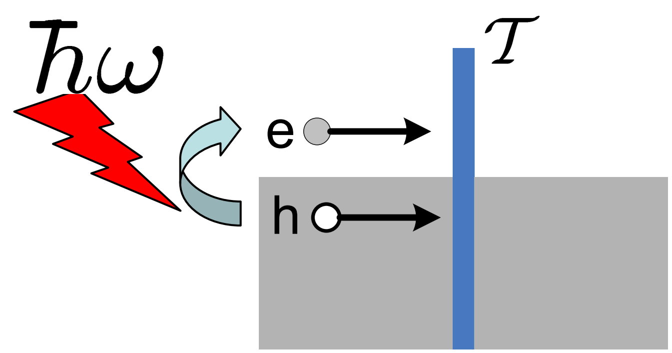
# Photo-assisted shot noise in dots

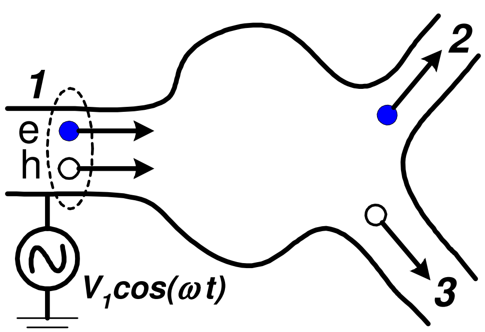
- Experiment on noise in QPC or quantum dots at low $T$

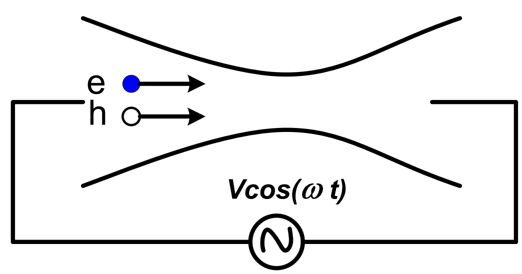
Reydellet et al'03

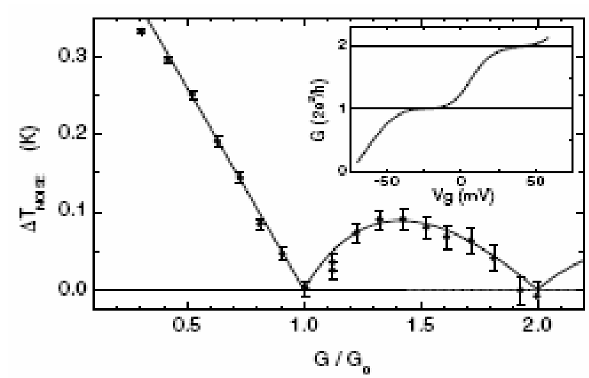
- Propose multi-terminal geometry to measure ***e-h*** correlation (noise)
- Controllable ***e-h*** correlations
- Scattering properties, interaction and decoherence are extracted from noise measurements

Electrons with energy $-\epsilon$ below $E_F$ excited to $\hbar\omega - \epsilon > 0$ leaves behind a hole $-\epsilon$



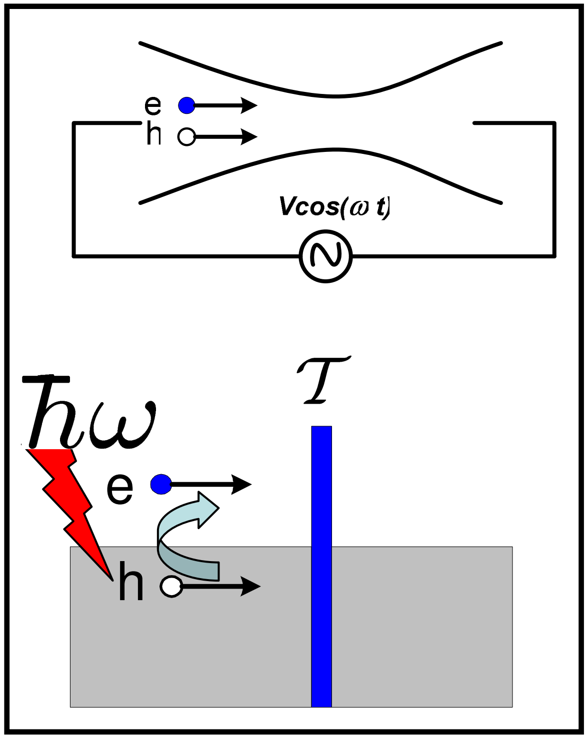

$$S \propto \sum_n T_n(1 - T_n)$$

# Two terminal geometry

$$\mathcal{P} = (eV)^2/(2\hbar\omega)^2 \ll 1$$

$$S^{ee} = \frac{\mathcal{P}e^2\omega}{2\pi}\sum_n T_n,\ S^{eh} = -\frac{\mathcal{P}e^2\omega}{2\pi}\sum_n T_n^2$$

$$S = 2(S^{ee} + S^{eh}) = \frac{\mathcal{P}e^2\omega}{\pi}\sum_n T_n(1 - T_n)$$




***e*** and ***h*** created in pairs pass independently

$$Q = n_e - n_h$$

If ***e-h*** transmissions are independent Poissonians (classic)

$$P(Q) = I_{2|Q|}(2\lambda T)\exp(-2\lambda T) \Rightarrow \langle\langle Q^2\rangle\rangle = 2\lambda T$$

**WRONG**

*N* pairs are created with $P(N) = e^{-\lambda}\lambda^N/N!$

Measured charge=difference of two binomial processes!

$$P(n|N) = C_N^n T^n (1 - T)^{N-n}$$

$$P(Q) \approx \frac{[\lambda T(1-T)]^{|Q|}}{|Q|!} \Rightarrow \langle\langle Q^2\rangle\rangle = 2\lambda T(1 - T)$$

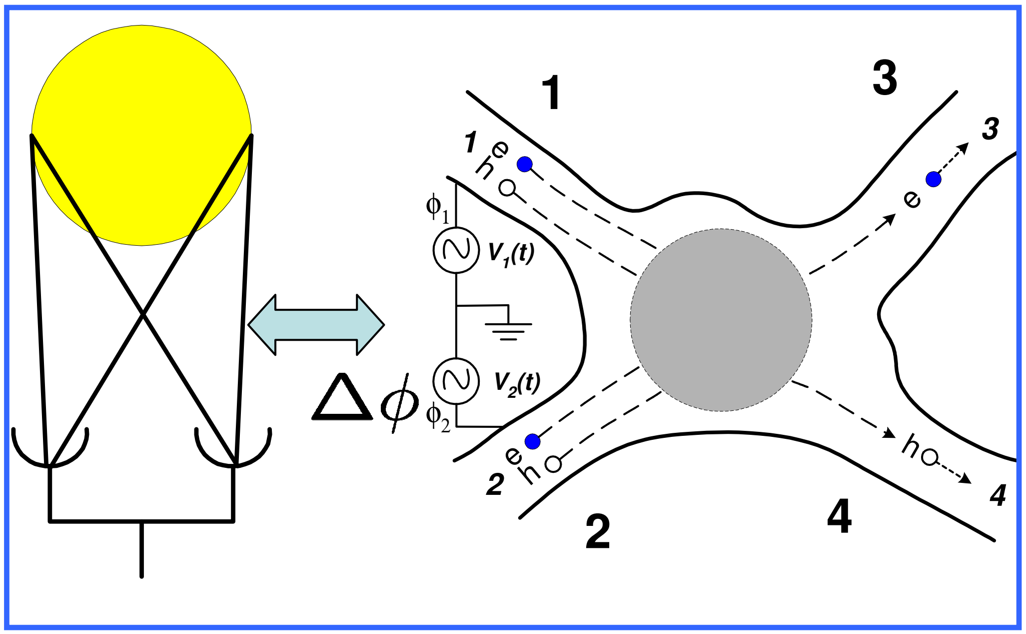
# HBT-phase

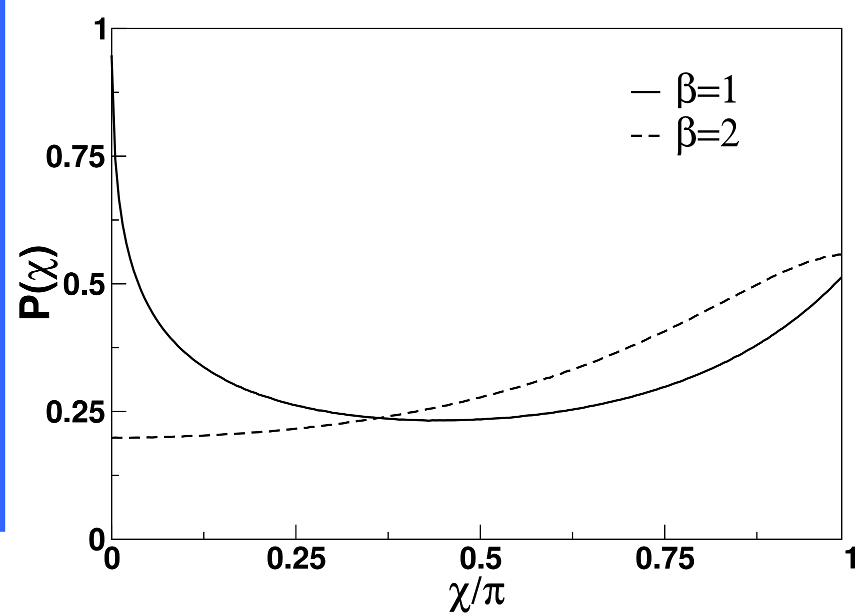
$$V_1(t) = V cos(\omega t) \ , \ V_2(t) = cos(\omega t + \Delta\phi)$$

HBT-interferometer. Intensity-intensity correlations from incoherent sources

$$S_{34}(\Delta\phi) = -\frac{e^2\omega}{2\pi}\mathcal{P}\left|s^\dagger_{13}s_{41}e^{-i\Delta\phi} + s^\dagger_{23}s_{42}\right|^2$$

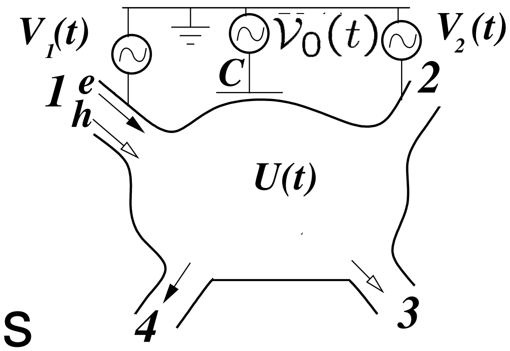
Extrema of noise at $\Delta\phi_+ = \chi \ , \ \Delta\phi_- = \Delta\phi_+ + \pi,$




Variation of $\Delta\phi$ controls the correlations. Single-channel dot has strongly non-uniform distribution of the HBT phase $\chi$ that maximizes the correlations (noise)

# Interactions and decoherence




- Averaged shot noise without interactions is

$$S_{\lambda\mu} = 2G_{\lambda\mu}k_{\mathrm{B}}T_0^*, \quad k_{\mathrm{B}}T_0^* = \frac{e^2}{\hbar\omega}\left(\frac{\mathrm{Tr}\ V^2}{N} - \frac{|\,\mathrm{Tr}\ Ve^{i\phi}|^2}{N^2}\right)$$

- Current and internal potential are found self-consistently

(dynamical charging is important at $\omega \neq 0$ )

$$k_{\mathrm{B}}T^* = k_{\mathrm{B}}T_0^* + \frac{e^2}{\hbar\omega}\frac{|\mathrm{Tr}\ (Ve^{i\phi})/N - V_0e^{i\phi_0}|^2(\omega\tau_{\mathrm{RC}})^2}{1+(\omega\tau_{\mathrm{RC}})^2}$$

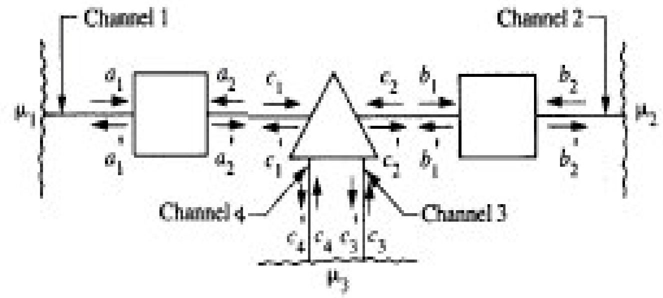
At GHz frequencies $\omega\tau_{\mathrm{RC}}$ -term becomes important

- Decoherence with rate $\hbar/\tau_\phi$ modifies noise, large decoherence kills correlations

$$T^* = \frac{T_0^* N\Delta}{N\Delta + \hbar/\tau_\phi}$$

# Decoherence from voltage probes

$$I_\alpha = \frac{e}{h}\left[(N_\alpha - R_{\alpha\alpha})\,\mu_\alpha - \sum_{\beta\neq\alpha} T_{\alpha\beta}\,\mu_\beta\right]$$




$$I_3 = 0\,; \Rightarrow V_3 = \frac{T_{31}V_1 + T_{32}V_2}{T_{31} + T_{32}} \Rightarrow I = I_1 = -I_2$$

$$G = \frac{e^2}{h}\left[T_{21} + \frac{T_{23}\,T_{31}}{T_{31} + T_{32}}\right]$$

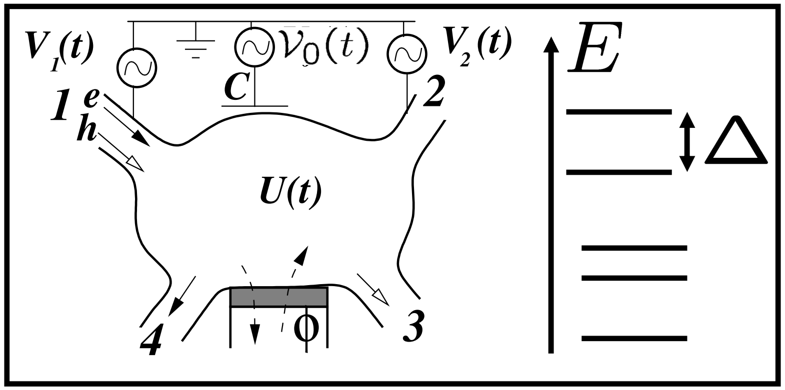
coherent incoherent

# Voltage probe+quantum dot

$$G = \frac{e^2}{h}\left[T_{21} + \frac{T_{23}T_{31}}{T_{31}+T_{32}}\right]$$

- Strength of inelastic processes and their locality is regulated by degree of coupling of voltage probe ( $N_\phi$ channels with transmission $\Gamma \in [0,1]$ )

Brouwer and Beenakker $\tau_\phi = \frac{h}{N_\phi \Gamma \Delta}$




- Limit of uniform decoherence:

$$N_\phi \to \infty, \Gamma \to 0, N_\phi \Gamma = \text{const}$$

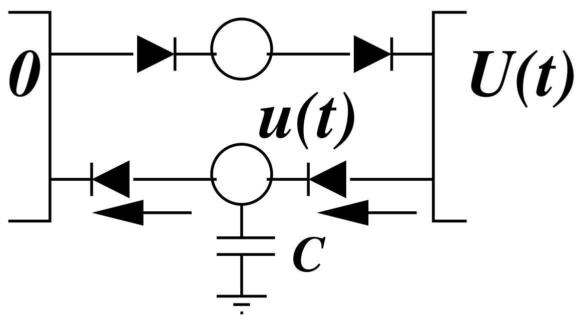
$$\langle G(\tau_\phi)\rangle = \langle G(\tau_\phi = 0)\rangle$$

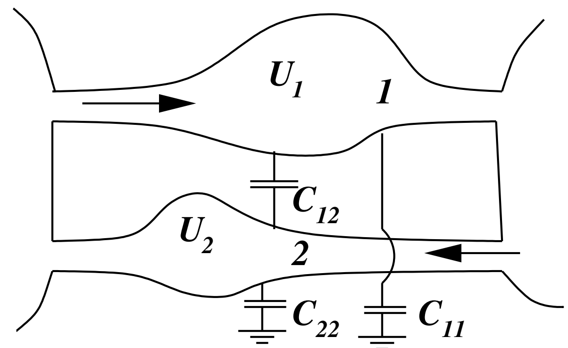
$$\tau_{\text{dwell}} = \frac{h}{N\Delta}$$

$$\delta G_{\text{WL}} = \frac{e^2}{h}\frac{N_1 N_2}{(N_1+N_2)^2} \cdot \frac{1}{1+\tau_{\text{dwell}}/\tau_\phi + (B/B_c)^2}$$

$$\text{var } G = \frac{2}{\beta}\left(\frac{e^2}{h}\frac{N_1 N_2}{(N_1+N_2)^2}\frac{1}{1+\tau_{\text{dwell}}/\tau_\phi}\right)^2$$

# Coulomb in AC-Quantum Hall




$$G_{\alpha\beta}(\omega) = dI_\alpha(\omega)/dV_\beta(\omega) \Rightarrow E_{\alpha\beta} = dG_{\alpha\beta}(\omega)/d\omega|_{\omega=0}$$

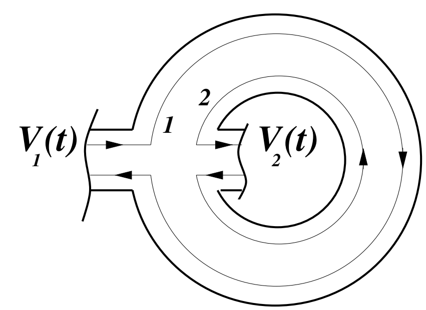
Charge build-up changes potentials

$\Rightarrow$ account of displacement currents restores gauge invariance and charge conservation

$$dI_k = \sum G_{kl} dV_l \qquad \sum_l G_{kl} = \sum_k G_{kl} = 0$$

If channels are close, Coulomb matters

$\Rightarrow$ modifies $G(\omega)$ in QH samples

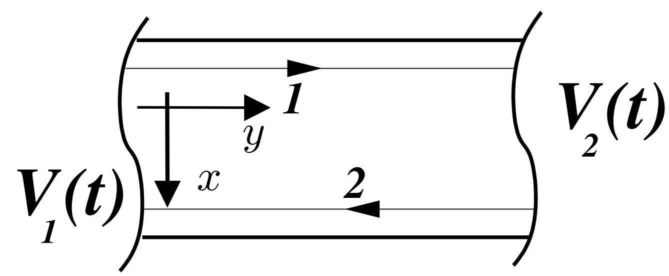


# How to conserve charge?

Self-consistency: charge build-up in channel *k* ⇨ raise internal potential $U_K$ ⇨

1)displacement current in *k*

2) induced currents in other channels

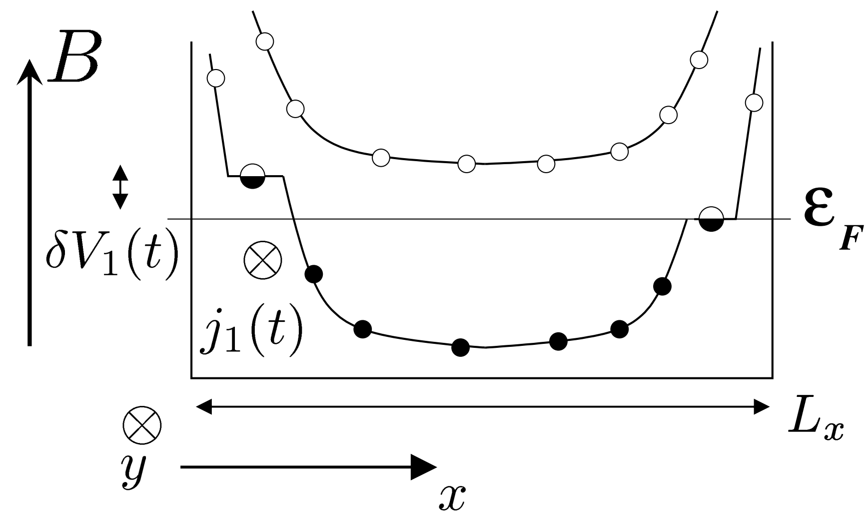



IQHE sample $\nu = 1$

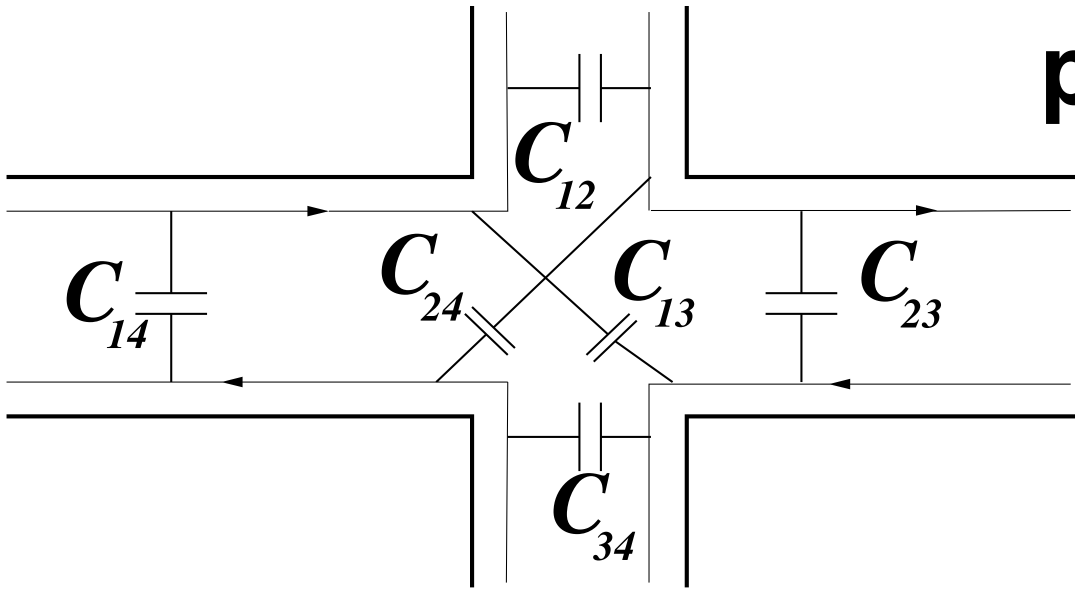
$$G_{\alpha\beta}(\omega) - G_{\alpha\beta}(0) \approx i\omega E_{\alpha\beta}$$

Find emittance $E_{\alpha\beta}$

between reservoirs $\alpha$ and $\beta$



# Step 1:Internal potentials



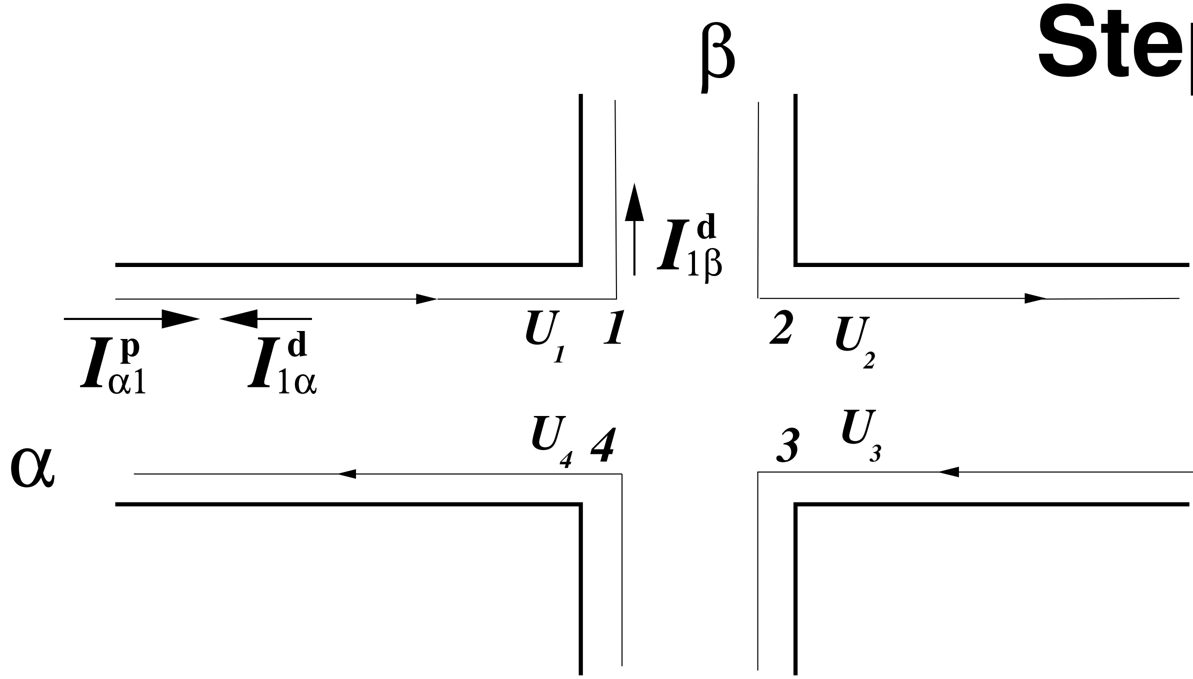

Assign internal potentials to every channel $\vec{U}(\omega) = (U_1, ..., U_4)^{\mathrm{T}}$

Gauge invariance is fulfilled by symmetric capacitance matrix $\hat{C}$

$$\sum_k C_{kl} = \sum_l C_{kl} = 0$$

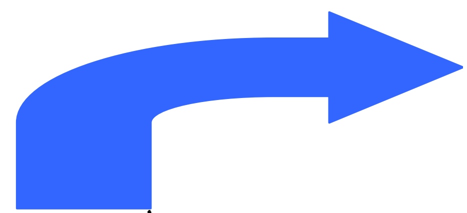
$$\vec{Q}(\omega) = \hat{C}\vec{U}(\omega) \Rightarrow \vec{I}(\omega) = i\omega\hat{C}\vec{U}(\omega)$$

# Step 2: Current balance

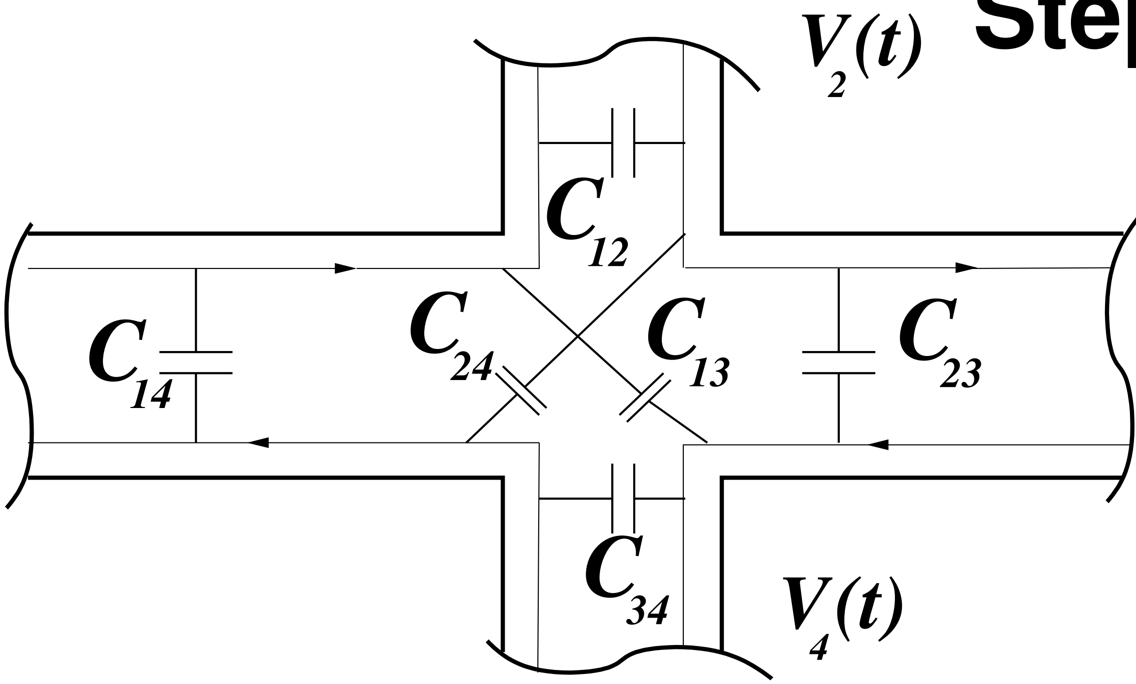



Total current $\vec{I}$, sum of particle currents $\hat{G}\vec{V}$ and displacement current $(-\hat{G}\vec{U})$ due to internal potentials, is gauge invariant. Knowing $\vec{V}$ we find $\vec{U}$

$$\vec{I}(\omega) = i\omega\hat{C}\vec{U}(\omega) = \hat{G}(\omega)(\vec{V}(\omega) - \vec{U}(\omega))$$



# Step 3: Set-up



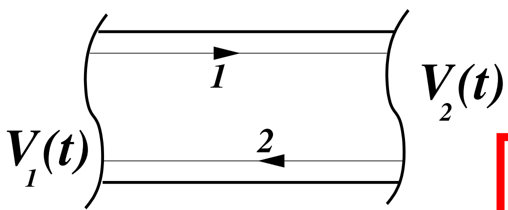

Final result is the matrix $\hat{C}_\mu$: response of charge in channels to variations of external voltages.

$$\vec{I}(\omega) = i\omega\hat{C}_\mu\vec{V}(\omega) = \boxed{\frac{1}{1+i\omega\hat{C}\hat{G}^{-1}(\omega)}}\hat{C}\vec{V}(\omega)$$

# Hall bar and Corbino disk

$$\vec{I}(\omega) = i\omega\hat{C}_\mu(\omega)\vec{V}(\omega) \Rightarrow E_{\alpha\beta} = \sum_{kl}(C_\mu(0))_{kl} \text{ } (\beta \text{ emits into } k, \, l \text{ injects into } \alpha)$$

$$E_{\alpha\beta}(B) = E_{\beta\alpha}(-B)$$




$$\hat{E} = -\hat{C}_\mu(0)$$

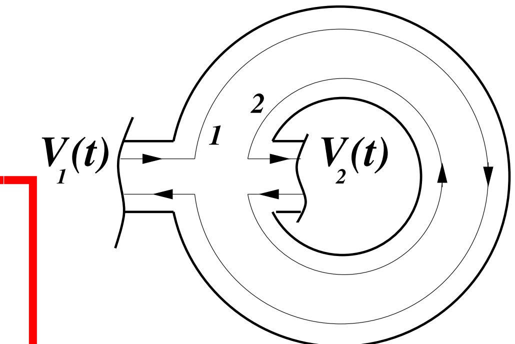
Inductive

$$\hat{C}_\mu(\omega) = C_\mu(\omega)\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$C_\mu(\omega) \approx C_\mu(0) + i\omega C_\mu^2(0)R_q$$

$$\frac{1}{C_\mu(0)} = \frac{1}{C} + \sum_{i=1}^{2}\frac{1}{e^2 dN_i/dE}$$

$$R_q = h/e^2$$




$$\hat{E} = \hat{C}_\mu(0)$$

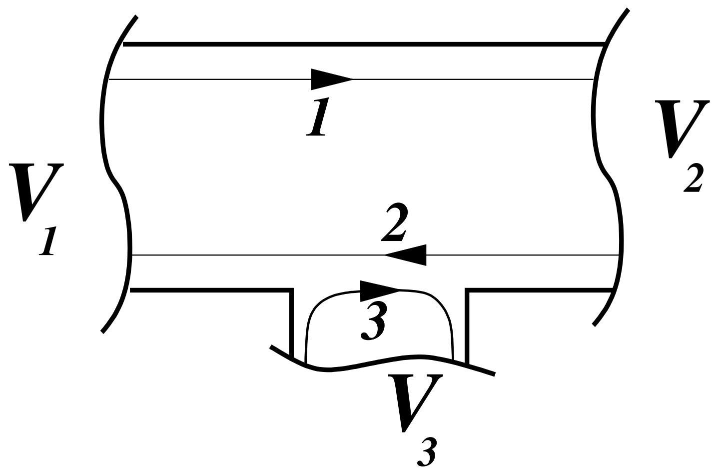
Capacitive

Symmetric to $B \to -B$: property of 2-channel system only

Capacitance: electrostatic $C+$ quantum $e^2 dN/dE \to$ electrochemical $C_\mu(0)$

Resistance standard $R_q$: $\omega^2$-term in AC conductance finds $e^2/h$

# Multi-terminal set-up




$$\hat{E} = \begin{pmatrix} C_{\mu,21} & C_{\mu,22} & C_{\mu,23} \\ C_{\mu,11} & C_{\mu,12} & C_{\mu,13} \\ C_{\mu,31} & C_{\mu,32} & C_{\mu,33} \end{pmatrix}$$

$$E_{13}(B) = C_{\mu,23},\ E_{13}(-B) = E_{31}(B) = C_{\mu,31} \neq E_{13}(B)$$

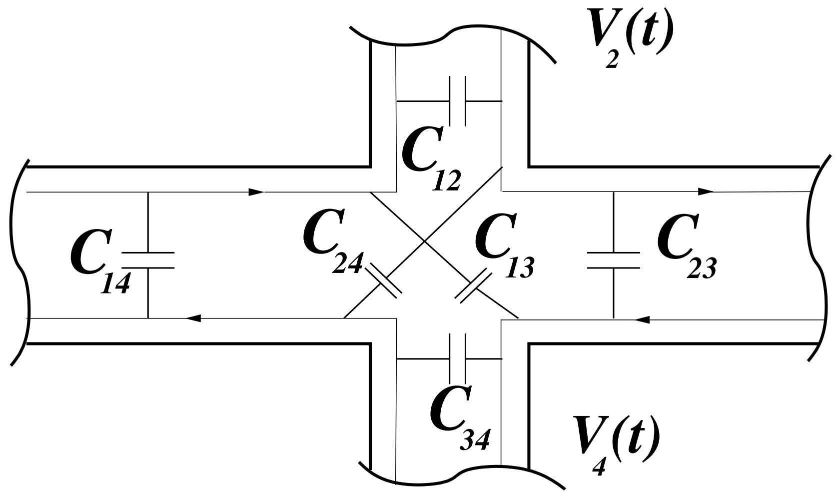
No symmetry as in the 2-terminal case

Explains magnetic asymmetry in experimental results

Chen et al ’94, Sommerfeld et al ‘96

# Longitudal and Hall resistance

$$
\begin{aligned}
G_{\alpha\beta} &= (e^2/h)(\delta_{\alpha,\beta} - \delta_{\alpha-1,\beta}) \\
E_{\alpha\beta} &= C_{\mu,\alpha-1,\beta} \\
R_L &= R_{12,34} = \frac{\delta V_3 - \delta V_4}{I_1}, \\
R_H &= R_{13,24} = \frac{\delta V_2 - \delta V_4}{I_1}
\end{aligned}
$$




Can fix $\delta V_4 = 0$ (gauge invariance) and $I_1 = I_2$ ( or $I_3$) for $R_L(R_H)$ set-up

$$
\begin{aligned}
R_L &= \frac{G_{32}G_{41} - G_{31}G_{42}}{D} \\
R_H &= \frac{G_{21}G_{43} - G_{41}G_{23}}{D}
\end{aligned}
$$

$D$ is $3 \times 3$ minor of $\hat{G}$

$$
\begin{aligned}
R_L &= i\omega \frac{C_{\mu,13}}{(e^2/h)^2}, \\
R_H &= \frac{h}{e^2} + i\omega \frac{C_{\mu,13} - C_{\mu,24}}{(e^2/h)^2}
\end{aligned}
$$

Can be checked in experiment

# Conclusions

- Coulomb interactions strongly modify AC conductance of QH sample

✓ Found quantum capacitance and resistance for 2-channel bar and disk.

✓ Explained conductance asymmetry $B \to -B$

✓ Found AC Hall and longitudal resistance

## ***Recommended Reading on Mesoscopic Transport:***

**Quantum Transport in Semiconductor Nanostructures,** C. W. J. Beenakker and H. van Houten, *Solid State Physics* ***44**, 1 (1991)* or *cond-mat/0412664*
(general review of 2D mesoscopics before 1991)

**Electronic transport in mesoscopic physics** S. Datta *(1995)*
(popular book on mesoscopics)

**Mesoscopic phenomena in solids,** B. Altshuler, P. A. Lee and R. A. Webb (eds), *Modern Problems in Condensed Matter Sciences* ***vol. 30*** *(1991)*

**Mesoscopic quantum physics,** E. Akermans, G. Montambaux, J.-L. Pichard and J. Zinn-Justin (eds), *(Les Houches 1994, Session LXI)*

**Mesoscopic Electron Transport,** L. L. Sohn, L. P. Kouwenhoven and G. Schon (eds) *NATO ASI Series E,* ***vol. 345*** *(1997)*

(reviews of modern mesoscopic experiments and theory)

**Shot Noise in Mesoscopic Conductors,** Ya. Blanter and M. Büttiker, *Phys. Rep.* ***336**, 1 (2000)* or *cond-mat/9910158* (general review of noise in solid state)

**Random-matrix theory of quantum transport,** C. W. J. Beenakker, *Rev. Mod. Phys.* ***69**, 731 (1997)* or *cond-mat/9612179* (theoretical review of mesoscopic dots, wires, and superconductors)